Application Criteria of Surface Impedance Boundary Conditions for Finite-Difference Time-Domain Analyses of Near- and Far-Field Configurations

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Abstract — An efficient implementation of the surface impedance boundary condition (SIBC) for the finite-difference time-domain (FDTD) method is presented in this paper. The surface impedance function of a lossy medium is approximated with a series of first-order rational functions by using the vector fitting (VF) technique. Thus, the resulting time-domain convolution integrals are efficiently computed using recursive formulas. A sensitivity analysis is performed to determine the minimum number of poles for several lossy media and field source configurations.

I. Introduction

Surface impedance boundary conditions (SIBCs) are an efficient way to analyze scattering from lossy dielectric objects or imperfect conductors by eliminating such regions from the solution domain [1]. This allows a large saving in computational resources. However, when the finite-difference time-domain (FDTD) method is used, a convolution integral in the time domain must be solved due to the frequency-dependent nature of the SIBC.

Recently, many researchers have implemented the SIBC using FDTD codes [2]-[4]. These several methods differs each other from the different way to efficiently evaluate such a convolution. In [2]-[3], the Prony's method was used to approximate the time-domain surface impedance by a series of exponential functions and therefore recursively evaluate the integral convolution. Specifically, a series of 20 exponential functions was employed in [2], while 10 terms only were adopted in [3]. Subsequently, Oh and Schutt-Aine approximated the normalized frequency-domain impedance with a first-order rational series expansion [4]. As well-known, rational functions in frequency-domain can be efficiently implemented with recursive methods in time-domain and a series of 6-8 rational functions were chosen to represent a good approximation in [4].

In this paper, the same approach of Oh and Schutt-Aine is undertaken but the normalized frequency-domain surface impedance is approximated by first-order rational functions coming from the vector fitting (VF) procedure [5] instead of a rational Chebyshev approximation routine. A sensitivity analysis is performed to verify the feasibility and accuracy of the proposed method (i.e., to determine the minimum number of poles) for several near- and far-field configurations and lossy media. The advantage of the proposed technique is that a reduced number of poles can be adopted for a limited frequency range and class of lossy media by retaining the same order of accuracy degree.

II. FDTD Implementation of SIBCs

The first-order (or Leontovich) impedance boundary condition in the frequency domain reads [1]

$$\boldsymbol{E}_t(\omega) = Z_c(\omega)\boldsymbol{n}\times\boldsymbol{H}_t(\omega) \qquad (1)$$

where subscript t denotes the tangential field components, $\boldsymbol{n}$ is the unit vector pointing outwards from the conducting body, and $Z_c(\omega)$ is the characteristic surface impedance of a medium having permeability μ_2, permittivity ε_2 and conductivity σ_2, given by

$$Z_c(\omega) = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} = \eta_2\sqrt{\frac{j\omega\varepsilon_2/\sigma_2}{1+j\omega\varepsilon_2/\sigma_2}}. \qquad (2)$$

To suitably perform the rational function approximation, the surface impedance function (2) is transformed into the Laplace domain and normalized [4]

$$Z_N(s') = \frac{1}{\eta_2}Z_c(\omega) = \sqrt{\frac{s'}{1+s'}} \qquad (3)$$

being $s' = j\omega/a$ and $a = \sigma_2/\varepsilon_2$.

By using the VF procedure, the normalized impedance (3) can be approximated by a rational function of the kind:

$$Z_N(s') \cong b - \sum_{i=1}^{L}\frac{C_i}{s'+p_i} \qquad (4)$$

where the coefficients b, C_i and p_i are respectively the gain term, and the i-th residue and pole extracted by the VF procedure, while L is the number of poles. Applying (4), the SIBC in the time domain can be written as

$$\begin{aligned}\boldsymbol{E}_t(t) &= Z_c(t)*\left[\boldsymbol{n}\times\boldsymbol{H}_t(t)\right] \cong \eta_2 b\left[\boldsymbol{n}\times\boldsymbol{H}_t(t)\right]\\ &-\eta_2\int_0^t\sum_{i=1}^{L}aC_i e^{-ap_i(t-\tau)}\left[\boldsymbol{n}\times\boldsymbol{H}_t(\tau)\right]d\tau\end{aligned}. \qquad (5)$$

Equation (5) can be efficiently implemented into FDTD codes using recursive convolutions. Also note that in (5) the poles and residues are fixed for any considered material. Thus, they are predetermined only once for any simulation.

III. Sensitivity Analysis

In [4], the normalized impedance (3) was approximated in the broad interval s' = [0, 3]. In this paper, the upper bound of the approximation interval is chosen to be s' = 0.1 since at this value correspond the frequency limit of good conductor (i.e., $\sigma_2/\omega\varepsilon_2 >> 1$). This is due to the applicability limits of the Leontovich impedance boundary condition (1). Moreover, despite of a unique approximation, two or more sub-intervals of s' are here investigated to minimize the number of poles. Indeed, when a limited frequency band is of interest, as for typical FDTD applications where the frequency range spans from some megahertzs to tens of gigahertzs, then a reduced interval of s' is required and the VF can be optimized only in that sub-interval. Obviously, the interval of s' depends on the values of a, therefore different bands can be selected for a lossy dielectric or imperfect conductors. The choice of the suitable band for a specific lossy media will be deeply described in the extended version of the paper, but the goodness of the proposed method is shown in Fig. 1, where the relative error of the normalized impedance is reported for several approximations. As compared with [4], for a fixed number of poles (i.e., L = 8) a more accurate solution is provided, especially for low values of s'. Moreover, when only a sub-interval of s' is desired, the number of poles can be reduced to L = 5 or 4 by retaining the accuracy degree.

To demonstrate the efficiency of the proposed method, the reflection coefficient $\Gamma = (Z_2 - Z_1)/(Z_2 + Z_1)$, where Z_1 is the wave impedance of the medium where the field is reflected and $Z_2 = Z_c$ is the wave impedance of the medium where the field is transmitted, is computed for three different propagation modes. Firstly, the reflection from a lossy media with σ_2 = 200 S/m, when propagating a plane wave in free space (Z_1 = 377 Ω), is considered (see Fig. 2). Then, the reflection from the same media when propagating an electric (Z_1 = 1000 Ω) or magnetic (Z_1 = 1 Ω) near-field is considered in Figs. 3 and 4, respectively. As can be observed, the importance of reducing the approximation error in the low band of s' is highlighted in the magnetic near-field propagation due to the low value of Z_1 which is comparable with the surface impedance Z_c. In that case, the SIBC adopted in [4] becomes unacceptable while the proposed one still follows the analytic results in the whole frequency range. Finally, if a reduced frequency range is of interest (e.g., the frequencies from some megahertzs to few gigahertzs), a good accuracy can be obtained with only L = 4 poles by using a VF weighted in the band named b2.

In the extended version of the paper more configurations will be deeper investigated and the VF coefficients will be explicitly given for the several bands.

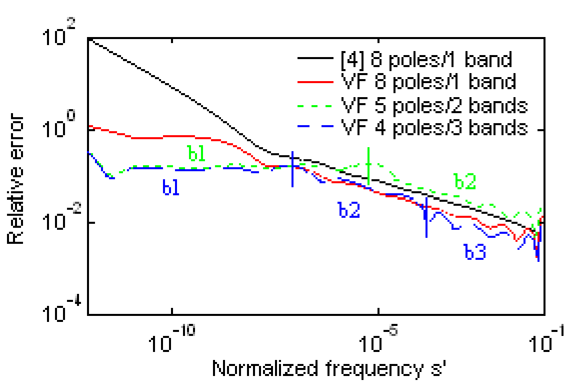

Fig. 1. Relative error of the normalized impedance for the several approximated rational functions.

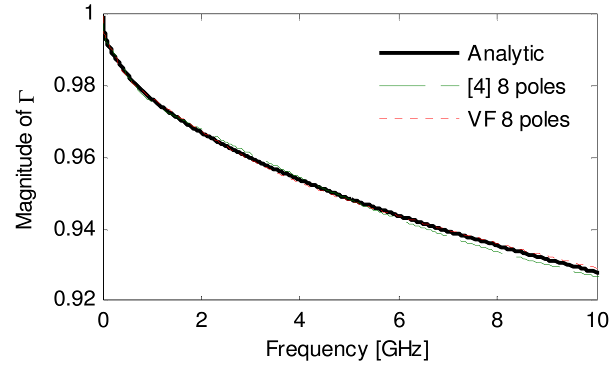

Fig. 2. Analytic and simulated reflection coefficients from a lossy dielectric with σ_2 = 200 S/m and far-field configuration (Z_1 = 377 Ω).

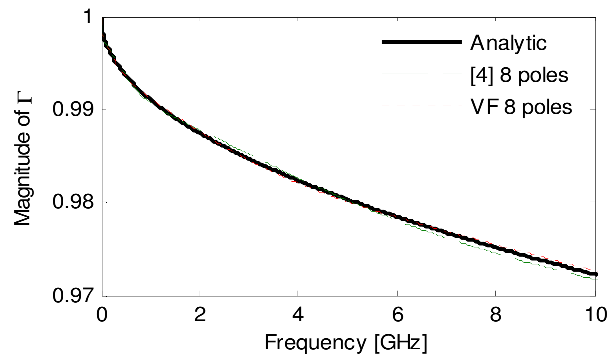

Fig. 3. Analytic and simulated reflection coefficients from a lossy dielectric with σ_2 = 200 S/m and near-field configuration (Z_1 = 1000 Ω).

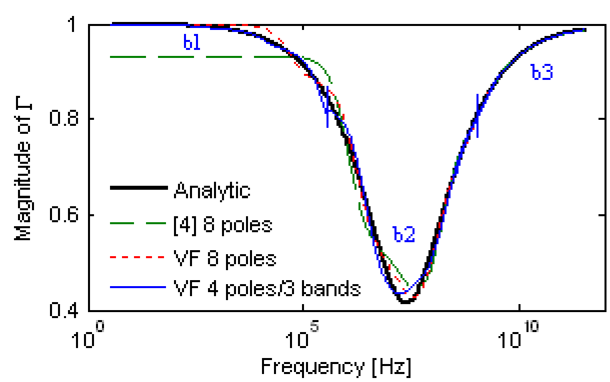

Fig. 4. Analytic and simulated reflection coefficients from a lossy dielectric with σ_2 = 200 S/m and near-field configuration (Z_1 = 1 Ω).

IV. REFERENCES

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